FLUKE®

Sources of Error in Time Interval Measurements

Application Note

Some timer/counters available today offer resolution of below one nanosecond in their time interval measurements. Of course, high resolution does not always guarantee high accuracy.
This application note explains the factors that contribute to inaccuracy in time interval measurements, and what steps you can take to reduce the inaccuracy of such measurements.
The inaccuracy of a measurement is quantified as uncertainty. Errors that account for these uncertainties are present in all time interval measurements, regardless of the counter used or how it is specified. The difference between counter models is the size of the errors.

Types of Measurement Uncertainties

Measurement uncertainties fall into two categories (see figure 1):

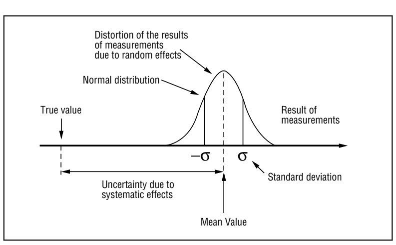

Figure 1. Graphical illustration of measurement uncertainty due to systematic and random effects.

- Uncertainty due to random effects, and
- Uncertainty due to systematic effects.

Errors due to random effects, of course, occur randomly, and the amount of the error varies in an unpredictable way each time you make a measurement. This error is often assumed to have an approximately normal distribution.
An error due to systematic effects, on the other hand, is unchanged when a measurement is repeated under the same conditions. It may, however, become evident whenever the test configuration is changed, or when environmental conditions change. This error causes an offset of the measurement result from the true value.
The following sections explain how to deal with both kinds of errors. Methods to reduce measurement uncertainties are also described.
Measurement Uncertainty due to Random Effects
Because errors due to random

effects vary from measurement to measurement, they will show up as unstable readings on the counter's display. These errors fall into two categories:
- Quantization error, and
- Trigger error due to noise.

Quantization Error
The Quantization Error (QE) is the single-shot time interval resolution of the measurement process used in the counter. This error is specified by its rms value, which is equal to the standard deviation (1σ) in a series of measurements. Fluke's PM 6680B offers a QE of 250 ps, and the PM 6681 offers a QE of 50 ps. The QE value is not to be confused with the unit value of the Least Significant Digit (LSD) displayed.

Trigger Error due to Noise
Trigger errors occur when a time interval measurement starts or stops too early or too late because of noise on the input signal. There are two sources of this noise:
- Noise on the signal being measured, and
- Noise added to this signal by the counter's input circuitry.

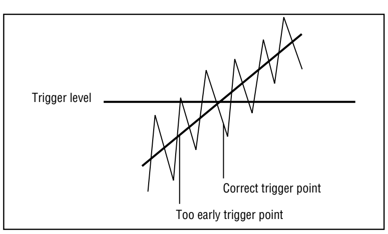

Figure 2. Noise on the input signal causes the counter to trigger too early.

As figure 2 shows, noise can cause a counter to trigger at the wrong time. The size of this error (expressed in time) is dependent on the amount of the noise and the slew rate of the signal.
Formula 1 is used to compute the standard deviation of both the start and stop trigger errors due to signal noise. On the Fluke PM 6680B and PM 6681, Vnoise-input is quite low - less than 200 µV and 100 µV, respectively.

$$\frac{\sqrt{(\text{Vnoise-input})^2 + (\text{Vnoise-signal})^2}}{\text{Signal slew rate (V/s) at trigger point}}$$

Formula 1. Start or Stop trigger error due to noise on the input signal.

Reducing Uncertainty due to Random Effects
You can reduce the quantization error - if the signal is not phase-locked to the counter's clock - by using averaging.

$$\frac{\sqrt{(\text{QE})^2 + (\text{Start Trigger Error}) + (\text{Stop Trigger Error})}}{\sqrt{N}}$$

or minimum: 1 ps for PM6681, 100 ps for PM6680B

Formula 2. Standard uncertainty due to random effects.

The trigger errors due to noise - as long as the noise behaves in a pseudo random manner - will also be reduced by averaging. Averaging will reduce these errors by the square root of the number of samples taken during the measurement. Formula 2 is used to compute the standard uncertainty due to random effects in an averaged measurement. ("N" denotes the number of samples in the measurement).
As an example, consider equal start and stop signals, which both have 5V p-p amplitude and 2 ns risetime (10% to 90%). This corresponds to a slew rate of 2V/ns. Assume 10 mV rms noise on the input signals. We calculate the start and stop trigger errors for PM 6680B using formula 1.

$$\frac{\sqrt{(0.2 \times 10^{-3})^2 + (10 \times 10^{-3})^2}}{2 \times 10^9} \times 5\ ps$$

Example. Calculation of the start or stop trigger error due to noise on the input signals.

We calculate the standard uncertainty with averaging over 100 samples using formula 2 as follows:

$$\frac{\sqrt{(250 \times 10^{-12})^2 + (5 \times 10^{-12})^2 \times (5 \times 10^{-12})^2}}{\sqrt{100}} \times 25\ ps \ (\text{or } minimum\ 100\ ps)$$

Example. Calculation of standard uncertainty due to random effects.

The interpolation technique used in PM 6680B and PM 6681 (also using large values of N) limits the standard uncertainty due to random effects to a minimum of 100 ps and 1 ps, respectively.

About Averaging
Both counters employ a measurement technique (HW-coded), which results in very fast averaged measurements (default set for all measuring functions) as follows:
- The PM 6680B automatically makes up to 1000 measurements/s, if the input signal frequency is >2 kHz.
- The PM 6681 automatically makes up to 6000 measurements/s, if the input signal frequency is >12 kHz.

A default measurement time that is useful for manual measurements is 0.2s. This results in up to 200 (for PM 6680B) and 1200 (for PM 6681) averaged measurements, which automatically reduces the quantization error to 100 ps, and to about 2 ps, respectively.

Measurement Uncertainty due to Systematic Effects
Unlike random uncertainty, which makes itself obvious by producing an unstable reading on the counter's display, systematic uncertainty does not produce unstable readings. Systematic uncertainty results from three sources:
- Trigger level timing error
- Channel mismatch error
- Timebase error.

Trigger Level Timing Error
This should not be confused with the trigger error due to noise on the input signal discussed previously. Trigger level timing error results from:
- Trigger level setting error due to deviation of the actual trigger level from the set (indicated) trigger level.

See figure 3.

- Input amplifier hysteresis if the input signals have unequal slew rates (PM 6680B only). See figure 4.

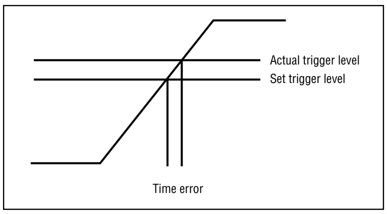

Figure 3. Trigger level timing error due to deviation of the actual trigger level from the set trigger level.

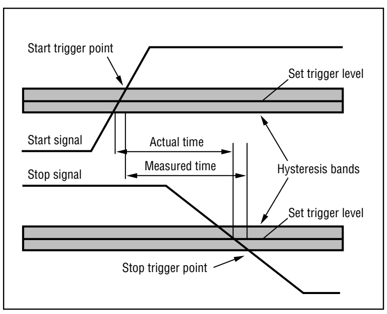

Figure 4. The hysteresis bands will cause errors in time measurements if the slew rates of the signals are different (PM 6680B only).

$$\frac{0.02^{1)} + 1\%\text{ of start trigger level (V)}}{\text{Slew rate (V/s) at start trigger point}} + \frac{0.02^{1)} + 1\%\text{ of stop trigger level (V)}}{\text{Slew rate (V/s) at stop trigger point}}$$

Formula 3. Trigger level setting error. 1) 0.02 for PM 6680B and 0.004 for PM 6681, or 0.2 and 0.04, respectively, if internal attenuator(s) have been turned on due to high input amplitude(s).

Formula 3 allows the amount of the trigger level setting error to be computed.
The value 0.02 (or 0.004 for PM 6681) in the formula is derived from the stability and the resolution of the DACs used to set the trigger levels. The 1% value refers to the accuracy of the reference voltage applied to the DACs.
Now, let us go back to the previous example and assume a trigger level of 2.5V for both inputs. The trigger level setting error is then calculated using formula 3 as follows:

$$\frac{0.02 + 0.01 \times 2.5}{2 \times 10^{9}} + \frac{0.02 + 0.01 \times 2.5}{2 \times 10^{9}} = 45\text{ ps}$$

Example.
Trigger level setting error calculation.

Input amplifiers employ a hysteresis band (see figure 4). The counter registers a positive transition only when the upper level of the band is passed. And a negative transition is registered only when the lower level of the band is passed.
The hysteresis band reduces errors in frequency measurements by eliminating incorrect triggering caused by noise. It can, however, cause an error in time interval measurements if the slew rates of the input signals are different for the start and stop channels. The hysteresis band on the PM 6680B is 30 mV, so the maximum trigger error in each trigger point is half of that or 15 mV.

$$\frac{0.015^{1)}}{\text{Slew rate (V/s) at start trigger point}} - \frac{0.015^{1)}}{\text{Slew rate (V/s) at stop trigger point}}$$

Formula 4. Input amplifier hysteresis error for PM 6680B. 1) 0.15 if internal attenuator(s) have been turned on due to high input amplitudes.

Formula 4 allows the input amplifier hysteresis error for PM 6680B to be calculated.
We have assumed equal slew rates for the start and stop channels in the example, so the input amplifiers' hysteresis errors cancel each other out. Compare with figure 4. The measured time and the actual time are equal when the slew rates of the signals are equal. The PM 6681 employs compensation for the hysteresis bands of the input amplifiers. The residual hysteresis error therefore becomes negligible.

Channel Mismatch Error

Channel mismatch error is a result of unequal propagation delays in the two channels of a timer/counter, and differences in risetimes of the input amplifiers. In Fluke's PM 6680B and PM 6681 these errors are specified as less than 1 ns and 500 ps, respectively. These errors are by far the largest sources of measurement uncertainty in measuring short time intervals (less than 1 µs).

Calibration of Channel Mismatch Error

Fortunately, the channel mismatch error can be reduced by performing a calibration of your measurement setup. To correct for this error, you can generate a "known time interval" between inputs A and B and note the difference between the measured and generated values. You can then use the value in either of two ways to correct the measurement:

- Use it as a correction term in measurements made by the controller in an automatic system, or
- Use it as a constant in the counter's own math function.

You can create a "known time interval" by using short cables, for example with one short cable (1 ns delay time) and one longer cable (3 ns delay time) plus a 50Ω power splitter. (You can order calibrated cables, model number PM 9588/01, and a power splitter, model number PM 9584/02, from Fluke).

Connect the input of the power splitter to your signal source, preferably a pulse generator with a fast (<2 ns) transition time. Connect the outputs of the power splitter to input A via the short cable and to input B via the longer cable. (See also figure 5). This provides a margin of safety against a negative time interval that could otherwise occur. The channel mismatch error can be corrected to about 200 ps for the PM

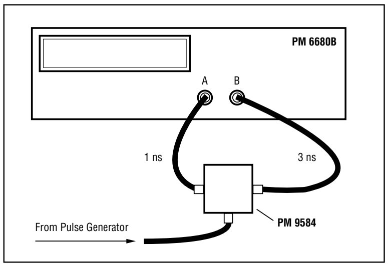

Figure 5. Setup for calibration of systematic error.

6680B, and to about 100 ps for the PM 6681, by careful measurement technique. Remember also to calibrate your cables between the measuring points and the counter's inputs in your measuring setup.

Timebase Error

An error is generated by deviations in the timebase frequency from its 10 MHz nominal value. Any change in frequency is directly translated into a time error in the measurement. Errors in the timebase results from two main sources:

- Aging of the oscillator, and
- Temperature changes.

Timebase	Aging/ year ε_A	Temp. ε_T
Standard	$<5\times10^{-6}$	$<1\times10^{-5}$
TCXO	$<5\times10^{-7}$	$<1\times10^{-6}$
High Stab. Oven	$<1\times10^{-7}$	$<1.5\times10^{-8}$
Very High Stab. Oven	$<7.5\times10^{-8}$	$<5\times10^{-9}$
Ultra High Stab. Oven	$<2\times10^{-8}$	$<2.5\times10^{-9}$
Rubidium	$<2\times10^{-10}$	$<3\times10^{-10}$

Table 1. Timebase options.

Table 1 shows these errors for the timebase oscillators from which you can select the one that best suits your require-ments. The errors are specified as relative errors with their maximum values. We can use formula 5 to calculate the standard deviation for the selected timebase.

$$\text{T.I.} \times \sqrt{\frac{\varepsilon_A^2 + \varepsilon_T^2}{3}}$$

Formula 5. Standard deviation of the timebase error due to aging and temperature variations.

In the example we use the standard timebase, with a one-year calibration cycle (max. error then equals the aging/year). If the time interval (T.I.) being measured is 1 µs, then the total timebase error would be:

$$10^{-6} \times \sqrt{\frac{(5 \times 10^{-6})^2 + (1 \times 10^{-5})^2}{3}} =$$

$$= 6.45 \times 10^{-12}\ \text{s} = 6.5\ \text{ps}$$

Example. Timebase error calculation.

If the measured time interval is short (less than 1 µs), then the timebase error is negligible compared with the channel mismatch error, also when using the standard timebase. However, the timebase error can become significant with longer time intervals if you need an uncertainty in measurement of better than about 10^{-5}.

Statement of Measurement Uncertainties

We have previously determined the different measurement errors in time interval measurements. But how do we state the numerical results of such measurements?

To combine the different errors, we must first translate them into quantities of the same kind. Manufacturers normally specify uncertainty due to random effects by the rms value (equal to the standard deviation). Uncertainty due to systematic effects, however, is mostly specified by the maximum values. The max. values (a_{max}) are considered to have a rectangular distribution. These can be translated into standard deviation by using formula 6.

$$\sqrt{\frac{\Sigma(a_{max})^2_i}{3}}$$

Formula 6.
Standard deviation calculated.

The combined standard uncertainty of a time interval measurement is calculated by formula 7. s_i stands for errors expressed in standard deviation and amax stands for errors expressed in max. values.

$$\sqrt{\Sigma s^2_i + \frac{\Sigma(a_{max})^2_i}{3}}$$

Formula 7.
Combined standard uncertainty.

Now we will use formula 7 to calculate the combined standard uncertainty in the example. We have already calculated the different errors with the following result:

- The uncertainty due to random effects is calculated to have a standard deviation of 25 ps, but the possible minimum value of 100 ps (specified for PM 6680B) has to be used.
- The trigger level setting error is calculated as 45 ps (a max. value).
- The input amplifier hysteresis error is 0.
- After calibration and correction, the channel mismatch error is 200 ps (amax. value).
- The timebase error is calculated as a standard deviation of 6.5 ps.

$$\sqrt{100^2 + \frac{45^2}{3} + \frac{200^2}{3} + 6.5^2}\ \text{ps} =$$

$$= 155\ \text{ps} = 0.16\ \text{ns}$$

Example. Combined standard uncertainty calculation for PM 6680B

The previous calculation results in a measure of uncertainty expressed as one standard deviation (1 s). The uncertainty can also be reported by the expanded uncertainty U = k x s, where k (the coverage factor) defines an interval estimated to have a level of confidence of:

- 68% with k = 1
- 95% with k = 2
- 99% with k = 3

You must also define the coverage factor in your document.

The result of a measurement of a time interval (measured to 1012.15 ns) can then be presented, for example, as:

1. (1012.15 ± 0.16) ns, with k=1.
2. (1012.15 ± 0.32) ns, with k=2.
3. (1012.15 ± 0.48) ns, with k=3.

You can also calculate the measurement errors when using the PM 6681 instead of the PM 6680B. This is done by making the same calculations for each error type as for the PM 6680B, but using the PM 6681's error figures. You will then get the following combined standard uncertainty:

$$\sqrt{5^2 + \frac{29^2}{3} + \frac{100^2}{3} + 6.5^2}\ \text{ps} =$$

$$= 60\ \text{ps} = 0.06\ \text{ns}$$

Example. Combined standard uncertainty calculation for PM 6681.

This gives a measurement uncertainty (1σ value) of 60 ps if you use the PM 6681 compared with 160 ps for the PM 6680B.

References

Users who are responsible for the statement of uncertainty in calibrations are recommended to refer to the now available guidelines, which establish general rules for evaluating and expressing uncertainty in measurements.

The following documents are recommended:

- Guide to the Expression of Uncertainty in Measurement. (ISBN 92-67-10188-9 1993)
- International vocabulary of basic and general terms in metrology.

Measuring Function	Uncertainty due to Random Effects (rms or standard deviation)	Uncertainty due to Systematic Effects (maximum values)
Time Interval Pulse Width Rise/Fall Time (s)	$\frac{\sqrt{(QE)^2 + (\text{Start Trigger Error})^2 + (\text{Stop Trigger Error})^2}}{\sqrt{N}}$ or min.: 1 ps for PM 6681, 100 ps for PM 6680B	± Timebase Error x Time Interval or Pulse Width or Rise/Fall Time ± Trigger Level Timing Error ± Δt Δt = Channel Mismatch Error Δt = 500 ps for PM 6681 Δt = 1 ns for PM 6680B
Frequency Period (Hz or s)	$\frac{\sqrt{(QE)^2 - 2x(\text{Start Trigger Error})^2}}{\text{Measuring Time}}$ x Frequency of Period	± Timebase Error x Frequency or Period
Ratio f_1/f_2	$\frac{\sqrt{(\text{Prescaler Factor})^2 + 2 \times (f_1 \times \text{Start Trigger Error of } f_2)^2}}{f_2 \times \text{Measuring Time}}$	
Phase (degrees)	$\frac{\sqrt{(QE)^2 + (\text{Start Trigger Error})^2 + (\text{Stop Trigger Error})^2}}{\sqrt{N}}$ x Frequency x 360° or min.: (1 ps for PM 6681, 100 ps for PM 6680B) x Frequency x 360°	± Trigger Level Timing Error x Frequency x 360° ± Δt x Frequency x 360° Δt = Channel Mismatch Error Δt = 500 ps for PM 6681 Δt = 1 ns for PM 6680B
Duty Factor	$\frac{\sqrt{(QE)^2 + (\text{Start Trigger Error})^2 + (\text{Stop Trigger Error})^2}}{\sqrt{N}}$ x Frequency or min.: (1 ps for PM 6681, 100 ps for PM 6680B) x Frequency	± Trigger Level Timing Error x Frequency ± Δt x Frequency Δt = Channel Mismatch Error Δt = 500 ps for PM 6681 Δt = 1 ns for PM 6680B

Table 2. Formulas for calculation of measurement uncertainties per measuring function.

(ISBN 92-67-01075-1 1993)
- Guidelines for the Expression of the Uncertainty of Measurement in Calibrations. (WECC Doc. 19-1990)

The first two documents can be ordered from the International Organization for Standardization in Geneva. The latest document is issued by the Western European Calibration Cooperation.

Summary of formulas
The formulas used for the previous calculations of the measurement uncertainties of time intervals are the basic ones. Formulas for other measuring functions are modified time interval formulas, converted into the specific function by multiplying by frequency, period, or frequency x 360.
Table 2 lists the formulas for calculation of measurement uncertainties for the different measuring functions.

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